# The Value of Equity Financing in Capital-Constrained Supply Chains under Different Carbon Regulations

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**Abstract:** With tightening environmental requirements, small and medium-sized enterprises (SMEs) may face the dual pressure of emission reduction and capital shortage. Focusing on a retailer-led supply chain in which the supplier is capital-constrained, we construct a Stackelberg game model to examine the equilibrium decision-making and value of equity financing. Our results show that capital constraints inhibit the supplier's emission-reduction effort, and the scarcer its own funds, the stronger this inhibition. Equity financing can relieve the capital shortage and incentivize higher emission reduction efforts. Under both cap-and-trade and carbon tax policies, equity financing admits a Pareto-improving region, yet its value is realized only when the dividend-sharing ratio is set appropriately. And, carbon parameters (the carbon price, tax rate) and the supplier's own funds jointly determine the boundary of this region. These findings offer theoretical implications for the operations and financing decisions of low-carbon supply chains.

**Keywords:** Cap-and-trade policy; Carbon tax policy; Equity financing; Stackelberg game

## 1. Introduction

With rapid economic development, environmental threats have emerged as a global risk. The IPCC states that climate change over the past five decades is very likely attributable to carbon emissions [1]. To curb such emissions, some countries and regions have adopted policy mechanisms, among which cap-and-trade and carbon tax policies are the most common. The Kyoto Protocol introduced a cap-and-trade policy that permits the trading of emission allowances in carbon markets [2,3], whereas a carbon tax levies a charge on carbon dioxide emissions [4]. Accordingly, carbon regulations reshape enterprises' cost structures, forcing them to reconsider the impact of carbon emissions when optimizing operational decisions.

In the context of low-carbon transition, capital constraints have become an unavoidable issue in supply chain operations. On the one hand, emission-reduction efforts require financial support, which compounds the difficulty for already capital-scarce SMEs, making funding shortages one of the primary obstacles to their willingness and enthusiasm for carbon abatement [5-7]. On the other hand, an individual enterprise's capital shortage can propagate along the supply chain and threaten the stability of the entire chain. It is therefore necessary to examine capital-constrained scenarios. Equity financing, favored for its low financial risk and ability to deepen cooperation, has expanded rapidly. In 2025, the total amount of equity financing in China exceeded 1 trillion yuan, a year-on-year

increase of 226.10%. In the context of low-carbon and environmentally friendly practices, whether equity financing can realize its value is an issue that deserves further investigation.

There is already a wealth of literature on low-carbon supply chain financing, but most of it focuses on bank credit and trade credit. Jiang et al. [8] examine the competition between platform credit financing and trade credit financing in e-commerce supply chains. Zhang and Kou [9] study bank-financing decisions under blockchain technology. Lu et al. [10] analyze product-line design under bank and trade credit financing. Aliabadi et al. [11] investigate supply chain decisions under promotional cost-sharing and three-level trade credit financing. In contrast, equity financing has received attention from only a few scholars. Liu et al. [12] explore operational decisions and coordination under carbon trading and equity financing, but their manufacturer-led model does not fit retailer-led supply chains such as Walmart's. Meanwhile, Yu and Wang [13] examine the choice between debt and equity financing for growth-stage firms, and Yan et al. [14] compare trade credit with equity financing under retailer loss aversion. However, neither considers carbon-reduction policies. Therefore, this paper incorporates both cap-and-trade and carbon tax policies and quantifies the value of equity financing under these two carbon schemes in a retailer-led supply chain.

Accordingly, this paper incorporates both cap-and-trade and carbon tax policies into a retailer-led Stackelberg game in which a capital-constrained supplier acts as the follower, derives the supply chain's equilibrium decisions under equity financing, and examines firms' operational and financing strategies. Specifically, it seeks to answer three questions: (1) Does the level of the supplier's own funds affect supply chain members' operational and financing decisions? (2) Does the retailer have an incentive to provide equity financing to the upstream supplier, and can the supplier obtain additional benefits from it? (3) What impact does equity financing have on firms' carbon emission reduction?

## 2. Model Description and Assumptions

Consider a retailer-led supply chain in which the supplier acts as the follower, producing and selling a single type of product. Carbon emissions arise during the production process, and the supplier can invest a carbon reduction effort $\Delta e$ to reduce the product's carbon emissions. Assume that market demand is given by $D = a - bp + g\Delta e$, where $a$ denotes the initial market demand, and $b$ and $g$ are the sensitivity coefficients of demand with respect to the retail price and carbon reduction effort, respectively. In addition, the supplier's carbon reduction effort incurs an abatement cost of $\eta\Delta e^2/2$, where $\eta > 0$ is the carbon reduction cost parameter, indicating an increasing marginal abatement cost.

In this Stackelberg game, the retailer first sets its marginal sales profit $m_R$, and the supplier then determines its marginal wholesale profit $m_S$ and carbon reduction effort $\Delta e$. However, because of a shortage of production funds, the supplier cannot achieve the optimal output and consequently fails to meet the retailer's optimal order quantity, imposing losses on the entire supply chain. The retailer therefore provides equity financing to the capital-constrained supplier. She injects funds according to the supplier's funding gap in exchange for a $\alpha$ proportion of equity. At the end of the selling season, the supplier does not need to repay principal or interest but must share dividends with the retailer at the negotiated ratio.

We further make the following assumptions: (1) the retailer has sufficient funds to meet its equity-investment needs; (2) the firm's downstream sales profit cannot be negative; (3) information is fully symmetric among supply chain members: market demand, and the supplier's initial production

funds are all shared.

For ease of exposition, the notations used in this paper and their definitions are summarized in Table 1. Superscripts $i = \{T, M\}$ denote the cap-and-trade and carbon tax policies, respectively, and superscripts $j = \{N, R\}$ denote the case of capital constraint without financial support and the case with equity financing, respectively.

**Table 1:** Notations and Definitions.

| Notation | Definition | Notation | Definition |
|---|---|---|---|
| $c$ | Unit production cost | $p_c$ | The initial market size |
| $e$ | Initial carbon emission per unit product | $z$ | Carbon tax rate |
| $a$ | Initial market size | $\Delta e^i$ | Carbon reduction effort |
| $b$ | The price impact factor | $m_S^i$ | The supplier's marginal sales profit |
| $g$ | The carbon reduction impact factor | $m_R^i$ | The retailer's marginal sales profit |
| $\eta$ | The carbon reduction cost parameter | $p^i$ | The retail price $p^i = c + m_S^i + m_R^i$ |
| $B$ | Supplier's own production funds | $\pi_R^i$ | The profit of the retailer under different carbon policies |
| $\alpha$ | The dividend ratio | | |
| $E$ | Carbon emission cap | $\pi_S^{ij}$ | The profits of the supplier under different carbon policies |

## 3. Supply Chain Decisions Under Cap-and-trade Policy

Under the cap-and-trade policy, the supplier faces an emission cap $E$. If its actual emissions fall below $E$, it can sell the surplus emission allowances in the market at the price $p_c$. If they exceed the cap, it must purchase additional allowances. For tractability, we assume that the buying and selling prices of emission allowances are equal and that all supply chain members maximize their profits.

### 3.1 The supplier has no financial constraints.

When the supplier is not capital-constrained, the supply chain members' decisions are not restricted by $B$, and the optimization problems of the supplier and the retailer can be formulated as follows:

$$\pi_S^T = m_S^T D^T - \frac{1}{2}\eta \Delta e^{T2} + p_c[E - (e - \Delta e^T)D^T] \quad (1)$$

$$\pi_R^T = m_R^T D^T \quad (2)$$

By backward induction, the optimal decisions of the supply chain members can be derived as $m_R^{T*} = \frac{a-bc-bep_c}{2b}$, $\Delta e^{T*} = \frac{(g+bp_c)(a-bc-bep_c)}{2[2b\eta-(g+bp_c)^2]}$, and $m_S^{T*} = \frac{-(a-bc)\eta+p_c[g(a+bc+2eg)-3be\eta]}{-2[2b\eta-(g+bp_c)^2]} + \frac{bp_c{}^2(a-bc+3eg+bep_c)}{-2[2b\eta-(g+bp_c)^2]}$, provided that the condition $B \geq \frac{cb\eta(a-bc-bep_c)}{2[2b\eta-(g+bp_c)^2]} = \bar{B}$ holds. Otherwise, the supplier faces a capital constraint, and the members cannot realize these optimal decisions.

### *3.2 The Supplier Faces Financial Constraints Without Financial Support*

When $B < \bar{B}$, the supplier's production funds are constrained, and it may be unable to obtain

external financial support. In this case, the supplier can only rely on its own funds to achieve a production quantity of $B/c$, i.e., $D^{TN*} = B/c$. Consequently, the retailer's marginal sales profit is subject to an upper bound, because the retailer must ensure that the supplier's profit exceeds $Ep_c$. Otherwise, the supplier would exit production and earn revenue by selling emission allowances. By backward induction, the supply chain members' decisions can be derived as follows:

$$\Delta e^{TN*} = \frac{B(g + bp_c)}{bc\eta} \tag{3}$$

$$m_S^{TN*} = \frac{2b^2e\eta cp_c + B(g^2 - b^2{p_c}^2)}{2b^2c\eta} \tag{4}$$

$$m_R^{TN*} = \frac{2bc\eta(a - bc - bep_c) - B[2b\eta - (g + bp_c)^2]}{b^2c\eta} \tag{5}$$

Substituting the derived results into Eqs. (1) and (2) yields the profits of the supplier and the retailer, $\pi_S^{TN*}$ and $\pi_R^{TN*}$, under the cap-and-trade policy without financial support.

Corollary 1. Under the cap-and-trade policy, the supplier's capital constraint inhibits its carbon reduction effort, and the stricter the constraint, the greater this inhibition.

Proof. From $B < \bar{B}$ we readily obtain $\Delta e^{TN*} - \Delta e^{T*} < 0$. Moreover, $d(\Delta e^{TN*} - \Delta e^{T*})/dB > 0$ indicates that as the supplier's own production funds decrease, the gap in abatement effort widens, implying a stronger negative effect.

### *3.3 Equity Financing Support for the Supplier*

To alleviate the negative effects of capital constraints on firms' operations, this section examines the case with equity financing. Under equity financing, the optimization problems of the supplier and the retailer can be formulated as follows:

$$\pi_S^{TR} = (1 - \alpha)\left\{(c + m_S^{TR})D^{TR} - B + p_c[E - (e - \Delta e^{TR})D^{TR}] - \frac{1}{2}\eta\Delta e^{TR2}\right\} \tag{6}$$

$$\pi_R^{TR} = m^{TR}D^{TR} - (cD^{TR} - B) + \alpha\pi_S^{TR} \tag{7}$$

The supplier's decision

Under equity financing, the supplier does not need to repay principal or interest but must share dividends with the retailer at the negotiated ratio. From Eq. (6), the supplier's optimal decisions can be derived.

Lemma 1. For a given $m_R^{TR}$, as long as $2b\eta - (g + bp_c)^2 > 0$, the supplier's optimal carbon reduction effort and wholesale price under equity financing are given by:

$$\Delta e^{TR*} = \frac{(g + bp_c)(a - bep_c - bm_R^{TR})}{2b\eta - (g + bp_c)^2} \tag{8}$$

$$m_S^{TR*} = \frac{\eta(a - 2bc - bm_R^{TR} + bep_c)}{2b\eta - (g + bp_c)^2} - \frac{[p_c(a - bc - bm_R^{TR} + eg) - cg](g + bp_c)}{2b\eta - (g + bp_c)^2} \tag{9}$$

From Lemma 1, we can get that $d\Delta e^{TR*}/dm_R^{TR} < 0$, whch implies that under equity financing, the retailer's marginal sales profit also adversely affects the supplier's abatement effort.

(1) The retailer's decision

Substituting the results of Lemma 1 into Eq. (7) and solving yields the retailer's optimal decisions.

Lemma 2. Under equity financing, the retailer's optimal marginal sales profit is:

$$m_R^{TR*} = \frac{(\alpha - 1)(a - bep_c) - bc}{b(\alpha - 2)} \tag{10}$$

From Lemma 2, we have $dm_R^{TR}/dp_c < 0$, which indicates that the retailer's marginal profit decreases as the carbon price increases. Note that under equity financing, the supplier's and the retailer's decisions are independent of the initial production funds $B$ but depend instead on the dividend-sharing ratio. Substituting the results of Lemmas 1 and 2 into Eqs. (6) and (7) yields the profits of the supplier and the retailer, $\pi_S^{TR*}$ and $\pi_R^{TR*}$, under the cap-and-trade policy with equity financing.

## 4. Supply Chain Decisions Under Carbon Tax Policy

Under the carbon tax policy, the supplier is charged a carbon tax at the rate $z$ per unit of carbon emission, and all supply chain members make decisions to maximize their profits.

### *4.1 The Supplier Has No Financial Constraints.*

If the supplier is not capital-constrained, the optimization problems of the supplier and the retailer can be formulated as follows:

$$\pi_S^M = m_S^M D^M - \frac{1}{2}\eta \Delta e^{M^2} - z(e - \Delta e^M)D^M \tag{11}$$

$$\pi_R^M = m_R^M D^M \tag{12}$$

By backward induction, the optimal decisions of the supplier and the retailer under the carbon tax policy are readily obtained as $m_S^{M*} = \frac{(g+bz)z(a+2eg-bc+bez)-(a-bc+3bez)\eta}{-2[2b\eta-(g+bz)^2]}$, $\Delta e^{M*} = \frac{(g+bz)(a-bc-bez)}{2[2b\eta-(g+bz)^2]}$ and $m_R^{M*} = \frac{a-bc-bez}{2b}$, provided that the condition $B \geq \frac{cb\eta(a-bc-bez)}{2[2b\eta-(g+bz)^2]} = \bar{\bar{B}}$ holds. If $B < \bar{\bar{B}}$, the supplier faces a capital constraint, and these optimal decisions cannot be realized.

### *4.2 The Supplier Faces Financial Constraints Without Financial Support*

Under the carbon tax policy, if $B < \bar{\bar{B}}$, the members' decisions are restricted; in this case, $D^{MN*} = B/c$, and by backward induction the supply chain members' equilibrium decisions can be derived as:

$$\Delta e^{MN*} = \frac{B(g + bz)}{bc\eta} \tag{13}$$

$$m_S^{MN*} = \frac{2b^2c\eta ez + B(g^2 - b^2z^2)}{2b^2c\eta} \tag{14}$$

$$m_R^{MN*} = \frac{2bc\eta(a - bc - bez) - B[2b\eta - (g + bz)^2]}{b^2c\eta} \tag{15}$$

Substituting the derived results into Eqs. (11) and (12) yields the profits of the supplier and the retailer, $\pi_S^{MN*}$ and $\pi_R^{MN*}$.

Corollary 2. Under the carbon tax policy, the supplier's capital constraint also inhibits its abatement effort, and the stricter the constraint, the greater this inhibition.

Proof. The proof is similar to that of Corollary 1 and is omitted for brevity.

### *4.3 Equity Financing Support for the Supplier*

Similarly, under the carbon tax policy, this section considers the case in which the supplier can obtain equity financing from the retailer. In this case, the optimization problems of the supplier and

the retailer can be formulated as follows:

$$\pi_S^{MR} = (1-\alpha)\left\{(c+m_S^{MR})D^{MR} - B - z[(e-\Delta e^{MR})D^{MR}] - \frac{1}{2}\eta\Delta e^{MR2}\right\} \tag{16}$$

$$\pi_R^{MR} = m_R^{MR}D^{MR} - (cD^{MR} - B) + \alpha\pi_S^{MR} \tag{17}$$

(1) The supplier's decision

Under equity financing, the supplier's optimal decisions can be derived from Eq. (16).

Lemma 3. For a given $m_R^{MR}$, as long as $2b\eta - (g+bz)^2 > 0$, the supplier's optimal abatement effort and wholesale price under equity financing are given by:

$$\Delta e^{MR*} = \frac{(g+bz)(a-bez-bm_R^{MR})}{2b\eta-(g+bz)^2} \tag{18}$$

$$\begin{aligned} m_S^{MR*} = \{&-(g+bz)[z(a-bc-bm_R^{MR}+eg)-cg] \\ &+\eta(a-bm^{MR}-2bc+bez)\}/[2b\eta-(g+bz)^2] \end{aligned} \tag{19}$$

(2) The retailer's decision

Substituting the results of Lemma 3 into Eq. (17) and solving yields the retailer's optimal decisions.

Lemma 4. Under equity financing, the retailer's optimal marginal sales profit is:

$$m_R^{MR*} = \frac{(\alpha-1)(a-bez)-bc}{b(\alpha-2)} \tag{20}$$

From Lemma 4, the retailer's decision incorporates the dividend-sharing arrangement. The larger the dividend-sharing ratio, the lower the retailer's marginal sales profit, reflecting that managers factor financing costs into their operational decisions. Substituting the results of Lemmas 3 and 4 into Eqs (16) and (17) yields the profits of the supplier and the retailer, $\pi_S^{MR*}$ and $\pi_R^{MR*}$, under the carbon tax policy with equity financing.

## 5. The Value of Equity Financing

This section takes the case of a capital-constrained supplier without financing support as the benchmark, compares equity financing schemes under different carbon regulations, and examines the value that equity financing creates for emission reduction and for each supply chain member.

### *5.1 The Cap-and-trade Policy*

(1) The value of equity financing for carbon emission reduction

Comparing the carbon reduction effort before and after financing yields:

Proposition 1. $\Delta e^{TR*} > \Delta e^{TN*}$.

Proof: According to formulas (3) and (8), we have

Proposition 1 indicates that under the cap-and-trade policy, equity financing enhances firms' capability and incentive for carbon emission reduction.

(2) The value of equity financing for the retailer

The retailer agrees to provide equity financing only when the value it derives from the equity financing scheme satisfies $\Delta\pi_R^{T-NR} = \Delta\pi_R^{TR*} - \Delta\pi_R^{TN*} \geq 0$.

Proposition 2. Under the cap-and-trade policy, if $p_c \geq \frac{a-3bc}{be} - \frac{2c}{(\sqrt{2}-1)e}$, the retailer is always willing to provide equity financing to the supplier. If $p_c < \frac{a-3bc}{be} - \frac{2c}{(\sqrt{2}-1)e}$, the retailer agrees to provide equity

financing only under the following conditions:

i) $0 \le B \le min\{B_1, b\eta c(a-bc-bep_c)/2H(p_c)\}$;

ii)$B_1 < B \le b\eta c(a-bc-bep_c)/2H(p_c)$ and $\alpha_R^{T-RN} \le \alpha \le 1$, where

$B_1 = \frac{1}{2H(p_c)}\{b\eta c[2(a-3bc-bep_c) - \sqrt{4(a-3bc-bep_c)^2 - 2(a-bc-bep_c)^2}]\}$, $\alpha_R^{T-RN}$satisfies $B(1-\alpha) + Ep_c\alpha + \frac{(a-bc-bep_c)^2}{2(2-\alpha)H(p_c)} - \frac{a-bc-bep_c}{b^2c^2\eta} + \frac{B^2H(p_c)}{2b^2c^2\eta} = 0$.

Proof. From Lemmas 1 and 2 and Eqs. (3)–(7), it readily follows that

$\Delta\pi_R^{T-RN} = -\frac{2Bb\eta c(a-bc-bep_c)-B^2H(p_c)}{2b^2c^2\eta} + (1-\alpha)B + Ep_c\alpha + \frac{\eta(a-bc-bep_c)^2}{2(2-\alpha)H(p_c)}$ and $\frac{d\Delta\pi_R^{T-RN}}{d\alpha} \ge 0$. When $\alpha = 1$, we have $\Delta\pi_R^{T-RN} > 0$. We now analyze the sign of $\Delta\pi_R^{T-RN}$ when $\alpha = 0$. When $\alpha = 0$, the discriminant of $\Delta\pi_R^{T-RN}$ with respect to $B$ is $\Delta = 8H^2(p_c)b^2c^2\eta^2[2(a-3bc-bep_c)^2 - (a-bc-bep_c)^2]$. Next, we discuss two cases:

(1) If $p_c \ge \frac{a-3bc}{be} - \frac{2c}{(\sqrt{2}-1)e}$, we have $\Delta \le 0$, in which case $\Delta\pi_R^{T-RN}(\alpha = 0) \ge 0$ always holds.

(2) If $p_c < \frac{a-3bc}{be} - \frac{2c}{(\sqrt{2}-1)e}$, we have $\Delta > 0$, indicating that the sign of $\Delta\pi_R^{T-RN}(\alpha = 0)$ is determined by $B$. Setting $\Delta\pi_R^{T-RN}(\alpha = 0) = 0$ yields $B_{1,2} = \{bc\eta[2(a-3bc-bep_c) \pm \sqrt{4(a-3bc-bep_c)^2 - 2(a-bc-bep_c)^2}]\}/2H(p_c)$ and $B_1 < B_2$. Since the supplier is capital-constrained, i.e., $B \le b\eta c(a-bc-bep_c)/2H(p_c) = \bar{B}$, a comparison readily yields $B_2 > \bar{B}$. When $B_1 \ge \bar{B}$, $\Delta\pi_R^{T-RN}(\alpha = 0) \ge 0$ always holds. When $B_1 < \bar{B}$, it can be further divided into several cases: (i) when $0 \le B \le B_1$, $\Delta\pi_R^{T-RN}(\alpha = 0) \ge 0$ always holds; (ii) when $B_1 \le B < \bar{B}$, $\Delta\pi_R^{T-RN}(\alpha = 0) < 0$, since $d\Delta\pi_R^{T-RN}/d\alpha \ge 0$ and $\Delta\pi_R^{T-RN}(\alpha = 1) > 0$, there exists a unique $\alpha_R^{T-RN}$ in [0,1] such that $\Delta\pi_R^{T-RN} = 0$, which ensures $\Delta\pi_R^{T-RN} > 0$ when $\alpha_R^{T-RN} \le \alpha \le 1$. By the above analysis, Proposition 2 follows.

Proposition 2 indicates that under the cap-and-trade policy, the value of equity financing depends not only on the dividend-sharing ratio but also on the carbon price and the supplier's own production funds. Specifically, if the carbon price exceeds the threshold, equity financing always generates additional value for the retailer; if it falls below the threshold, equity financing creates additional value for the retailer only when the supplier is severely capital-constrained or the dividend-sharing ratio is sufficiently large.

(2) The value of equity financing for the supplier

Similarly, the supplier accepts the equity financing scheme only when the value it derives from equity financing satisfies $\Delta\pi_S^{T-RN} = \Delta\pi_S^{TR*} - \Delta\pi_S^{TN*} \ge 0$.

Proposition 3. Under the cap-and-trade policy, if $0 \le \alpha \le \alpha_S^{T-RB}$, the supplier accepts equity financing, where $\alpha_S^{T-RN}$ satisfies $\Delta\pi_S^{T-RN}$=0.

Proof. From Lemmas 1 and 2 and Eqs. (3)–(7), we have $d\Delta\pi_S^{T-RN}/d\alpha < 0$. When $\alpha$=0, $\Delta\pi_S^{T-RN} \ge 0$; when $\alpha$=1, $\Delta\pi_S^{T-RN} = -Ep_c < 0$. Hence, there exists a unique $\alpha$=1 in [0,1] such that $\alpha$=1. When $0 \le \alpha \le \alpha_S^{T-RB}$, $\Delta\pi_S^{T-RN} \ge 0$.

Proposition 3 indicates that the supplier accepts the equity financing scheme only when the dividend-sharing ratio is below the threshold. In addition, $d\Delta\pi_S^{T-RN}/dB < 0$ shows that the stricter the supplier's capital constraint, the greater the additional value equity financing creates for the supplier and the stronger its incentive to accept equity financing.

(4) The Pareto-improving region

Analyzing the Pareto-improving region in which both the retailer and the supplier accept equity financing yields the following result.

Corollary 3. Under the cap-and-trade policy, if $p_c \geq \frac{a-3bc}{be} - \frac{2c}{(\sqrt{2}-1)e}$ or $0 \leq B \leq min\{B_1, b\eta c(a - bc - bep_c)/2H(p_c)\}$, equity financing achieves a Pareto improvement when $\alpha_R^{T-RB} \leq \alpha \leq \alpha_S^{T-RB}$. If $p_c < \frac{a-3bc}{be} - \frac{2c}{(\sqrt{2}-1)be}$ and $B_1 < B \leq b\eta c(a - bc - bep_c)/2H(p_c)$, equity financing achieves a Pareto improvement when condition $min\{\alpha_R^{T-RB}, \alpha_R^{T-RN}\} \leq \alpha \leq \alpha_S^{T-RB}$ is satisfied.

Corollary 3 provides a Pareto-improving region in which both the retailer and the supplier accept equity financing. Although the carbon price and the supplier's own production funds directly shape the boundary conditions of this region, a robust pattern holds: the dividend-sharing ratio required by the retailer cannot be too low, while that acceptable to the supplier cannot be too high. To illustrate the Pareto-improving effect of equity financing under the cap-and-trade policy, we present a numerical example. Figure 1 shows that the retailer provides equity financing only when $\alpha > 0.52$, whereas the supplier obtains value from equity financing only when $\alpha < 0.77$. For equity financing to be realized, the dividend-sharing ratio must be neither too high nor too low, and can be set only within the blue hatched region.

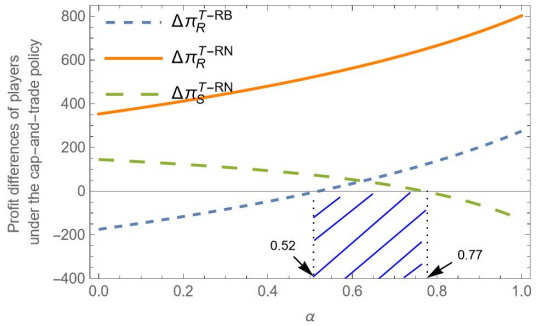



**Figure 1:** The Pareto-improving region of equity financing under the cap-and-trade policy.

### *5.2 The carbon tax policy*

(1) The value of equity financing for carbon emission reduction

Comparing the abatement effort before and after financing yields:

Proposition 4. $\Delta e^{MR*} > \Delta e^{MN*}$

Proof. The proof is analogous to that of Proposition 1 and is omitted for brevity.

Proposition 4 indicates that, under the carbon tax policy, equity financing also enhances firms' capability and incentive for emission reduction, consistent with the finding under the cap-and-trade policy.

(2) The value of equity financing for the retailer

Similar to the cap-and-trade policy, under the carbon tax policy, the retailer agrees to provide equity financing only when $\Delta\pi_R^{M-RN} = \Delta\pi_R^{MR*} - \Delta\pi_R^{MN*} \geq 0$.

Proposition 5. Under the carbon tax policy, if $z \geq \frac{a-2bc}{be} - \frac{c}{(\sqrt{2}-1)e}$, the retailer is always willing to provide equity financing to the supplier. If $z < \frac{a-2bc}{be} - \frac{c}{(\sqrt{2}-1)e}$, the retailer agrees to provide equity financing only under the following conditions:

(i) $0 \le B \le min\{B_3, b\eta c(a-bc-bez)/2H(z)\}$; (ii) $B_3 < B \le b\eta c(a-bc-bez)/2H(z)$ and $\alpha_R^{M-RN} \le \alpha \le 1$, where $B_3 = \frac{1}{2H(z)}\{b\eta c[2(a-2bc-bez) - \sqrt{4(a-2bc-bez)^2 - 2(a-bc-bez)^2}]\}$ and $\alpha_R^{M-RN}$ satisfy $B(1-\alpha) - \frac{2Bb\eta c(a-bc-bez) - B^2H(z)}{2b^2c^2\eta} + \frac{(a-be-bez)^2}{2(2-\alpha)H(z)} = 0$.

Proof. The proof is analogous to that of Proposition 2 and is omitted for brevity.

Proposition 5 indicates that, under the carbon tax policy, whether equity financing creates additional value for the retailer depends on the carbon tax rate, the supplier's own production funds, and the dividend-sharing ratio. Specifically, if the carbon tax rate is high, equity financing always creates value for the retailer; if the tax rate is low, it creates additional value only when the supplier is severely capital-constrained or the dividend-sharing ratio is sufficiently large.

(3) The value of equity financing for the supplier.

Similarly, under the carbon tax policy, the supplier can obtain additional value from the equity financing scheme when $\Delta\pi_S^{M-RN} = \Delta\pi_S^{MR*} - \Delta\pi_S^{MN*} \ge 0$.

Proposition 6. Under the carbon tax policy, the supplier is always willing to accept equity financing.

Proof. From Lemmas 3 and 4 and Eqs. (13)–(17), we have $d\Delta\pi_S^{M-RN}/d\alpha \le 0$, and $\Delta\pi_S^{M-RN}(\alpha = 1) = 0$. Hence, $\Delta\pi_S^{M-RN} \ge 0$ holds on [0,1].

Proposition 6 indicates that under equity financing, the supplier's profit is always no lower than that in the case without financial support, because equity financing induces the retailer to prioritize mutual benefit, thereby increasing the supplier's profit.

(4) The Pareto-improving region

For the Pareto-improving problem under the carbon tax policy in which both the retailer and the supplier accept equity financing. According to propositions 3 and 4, the following conclusions can be drawn.

Corollary 4. Under the carbon tax policy, if $z \ge \frac{a-2bc}{be} - \frac{c}{(\sqrt{2}-1)e}$ or $0 \le B \le min\{B_2, b\eta c(a-bc-bez)/2H(z)\}$, equity financing achieves a Pareto improvement when condition $\alpha_R^{M-RB} \le \alpha \le 1$ is satisfied. If $z < \frac{a-2bc}{be} - \frac{c}{(\sqrt{2}-1)e}$ and $B_2 < B \le \frac{b\eta c(a-bc-bez)}{2H(z)}$, equity financing achieves a Pareto improvement when condition $min\{\alpha_R^{M-RB}, \alpha_R^{M-RN}\} \le \alpha \le 1$ is satisfied.

Corollary 4 provides the Pareto-improving region under the carbon tax policy in which both the retailer and the supplier accept equity financing, and the retailer consistently requires the dividend-sharing ratio not to be too low. Figure 2 illustrates how each member's profit difference varies with $\alpha$. Specifically, equity financing creates value for the retailer only when $\alpha \ge 0.17$; otherwise, the retailer has no incentive to provide equity financing. For the supplier, the value of equity financing disappears when $\alpha = 1$, whereas the supplier always benefits from accepting equity financing when $\alpha < 1$.

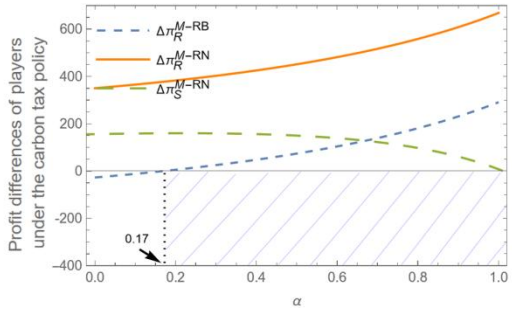



**Figure 2:** The Pareto-improving region of equity financing under the carbon tax policy.

## 6 Conclusion

Against the backdrop of equity financing and the capital constraints faced by SMEs under carbon regulations, we solve the optimal operational decisions and profits of supply chain members under both the cap-and-trade and carbon tax policies, and examine the value of equity financing in promoting carbon emission reduction and enhancing members' profits. Our main findings are as follows: (1) capital constraints inhibit the supplier's emission-reduction effort, and the stricter the constraint, the stronger this inhibition; (2) equity financing incentivizes the supplier to exert greater abatement effort, which benefits the environment; (3) under both policies, equity financing admits a Pareto-improving region, yet its value is conditional, and a well-designed dividend-sharing ratio is key to unlocking this value and achieving a win-win outcome for the supply chain.

These findings offer theoretical references for capital-constrained supply chain firms under carbon regulations in making operational and financing decisions. Nevertheless, this study is based on a dyadic (single-supplier – single-retailer) supply chain; future work could extend the analysis to a competitive environment with two suppliers or two retailers.

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