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Analysis of Stiffness Reduction Coefficient of Hollow Slab Girder Bridges Based on Crack Parameters

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Abstract: Concrete hollow slab beam bridges are an important part of the highway transportation network. Cracks that occur under long - term loads can alter the stiffness of the beam bridges and affect their structural performance. This paper takes a single 13 - meter simply - supported hollow slab beam as the research object and constructs a finite - element model with a mesh size of 0.1m. The relationships between parameters such as the length, depth, position, and number of load - induced cracks on the bottom slab and the stiffness reduction coefficient $\boldsymbol{\mu}$ were studied. The results show that the stiffness reduction coefficient is inversely proportional to the crack length and height. Cracks at the mid - span have a greater impact on $\boldsymbol{\mu}$, and the stiffness loss caused by multiple cracks is similar to the sum of the stiffness losses caused by single cracks. Through the analysis of the normal stress loss at the cracks and combined with relevant theories, a relational expression between the stiffness reduction coefficient and crack characteristics was obtained. After simulation calculations for three types of simply - supported hollow slab beams (8m, 13m, and 16m), the average error between the stiffness reduction coefficient calculated by the formula in this paper and the value calculated by the finite - element method is 1.43%. This provides a valuable reference for the research on the stiffness reduction of concrete beam bridges.

Keywords: Hollow slab girder bridge; Crack parameters; Stiffness reduction coefficient; Finite-element model

1. Introduction

According to the bridge types of a certain ring - expressway in a city, hollow slab girders account for 44% of all the bridges on the ring - expressway. As a key component of the highway transportation network, the structural performance of concrete hollow slab girder bridges is closely related to the safety and durability of the bridges. During long - term use, cracks caused by loadings are inevitable, which can significantly change the stiffness of the girder bridges, weaken their bearing capacity and service performance. Therefore, exploring the stiffness reduction law of concrete girder bridges based on load - induced cracks has important theoretical and practical significance.

D.K. Agarwalla [1] studied the influence of vibration cracks of a cantilever beam on modal parameters. The crack was simulated as a mass - less spring to determine the natural frequency, and a comprehensive analysis was carried out by combining finite - element numerical methods with an experimental device equipped with accelerometers, opening up a new way for studying the influence of cracks on the modal of structures. Yongjun Zhou [2] et al. conducted full - scale experiments on the static and dynamic stiffness of prestressed concrete girders. They selected three old - bridge girders, determined the equivalent flexural and dynamic stiffness algorithms, and obtained the changes in loading stiffness, crack frequency sensitivity, and the influence of boundary conditions through static and dynamic tests. K. Vigneshwaran and R.K. Behera [3] explored the dynamic characteristics of simply - supported beams with multiple breathing cracks. Through theoretical derivations and the finite - element method, they obtained the beam stiffness matrix and motion equations based on influence coefficients, etc., and carried out numerical calculations and compared the parameters of different cracked beams.

These research results reveal the influence mechanism of cracks on bridge performance from different aspects. This paper starts with a detailed analysis of the stiffness reduction law from the aspects of the length, depth, and number of load - induced cracks on the bottom slab, and finally obtains the calculation relationship between the crack characteristics of the bottom slab of the hollow slab girder and the stiffness reduction coefficient.

2. Analysis of the Relationship between Crack Characteristics and μ

This paper takes a single 13 - meter simply - supported hollow slab beam as an example for research. The material property is that of C40 concrete. According to the literature research [4][5], the smaller the mesh size of the finite - element model, the closer the research results of the model are to the experimental results. However, the smaller the mesh size, the higher the requirements for computer performance and the longer the running time. Therefore, after comprehensive consideration, the mesh size in this paper is set to 0.1m. This can not only make the test results more accurate but also avoid spending a large amount of time on running the analysis and consuming a large amount of computer resources. The finite - element model is shown in Figure 1.

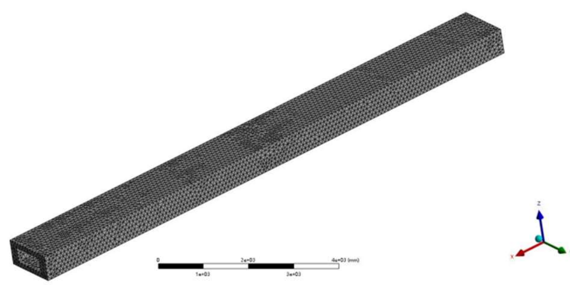


Figure 1: Finite Element Model of Simply Supported Hollow Slab.

According to the General Specifications for Design of Highway Bridges and Culverts (JTG D60 - 2015), the approximate formula for the natural vibration frequency (fundamental frequency) of a simply supported beam bridge is as follows:

$$f = \frac{\pi}{2l^2}\sqrt{\frac{EI_c}{m_c}} \tag{1}$$

l represents the calculated span of the structure; E represents the elastic modulus of the structural material; I_c represents the moment of inertia of the cross - section at the mid - span of the structure; m_c represents the mass per unit length at the mid - span of the structure.

Then the stiffness reduction coefficient is:

$$\mu = \frac{(EI_c)^*}{EI_c} = \frac{f^{*2}}{f^2} \tag{2}$$

f^* represents the fundamental frequency of the bridge after cracks occur.

Load - induced cracks mostly occur due to the bending of the bridge during its service stage, resulting in transverse cracks at the bottom. This paper studies the relationships between the length, height, position, and number of transverse cracks and μ. Firstly, the relationships between the length and height of a single crack and μ are investigated. The crack is located at the mid - span of the bottom slab, and the crack length extends towards the web. The results are shown in Figure 2.

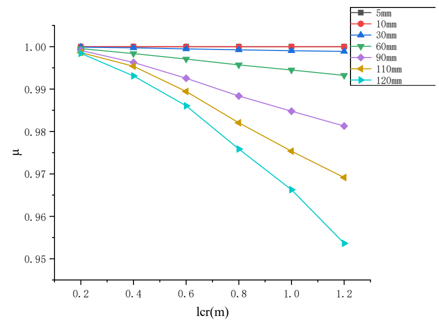


Figure 2: Relationship Between Crack Length and μ at Different Crack Heights.

From the above figure, it can be seen that the stiffness reduction coefficient is inversely proportional to both the crack length and height, and it continuously decreases as the crack length and height increase. By adjusting the longitudinal position of the crack, the influence of the crack's longitudinal position on the stiffness reduction coefficient was explored. With a crack height of 60mm and the length extending from the transverse center towards the web, the results are shown in Figure 3.

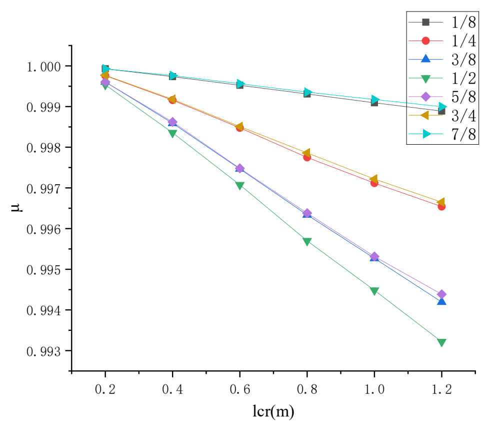


Figure 3: Relationship Between Crack Length and μ at Different Positions.

As can be seen from Figure 3, the closer the crack is to the mid - span, the greater its impact on μ. The closer the crack is to the side - span, the smaller the value of μ. Cracks at the same positions on both sides of the mid - span have the same impact on μ. To study the influence of different numbers of transverse cracks on the stiffness reduction coefficient, with a crack height of 60mm, the number of cracks was increased with a uniform distribution in the longitudinal position of the bridge. The positions are shown in

Table **1**, and the crack length extends towards the bottom slab. The results are shown in Figure 4.

Table 1: Table of Transverse Crack Positions on the Bottom Slab.

Crack Quantity	1	3	5	7	9
Longitudinal Position	1/2	1/4, 1/2, 3/4	1/6, 1/3, 1/2, 2/3, 5/6	1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8	1/9, 2/9, 1/3, 4/9, 5/9, 2/3, 7/9, 8/9

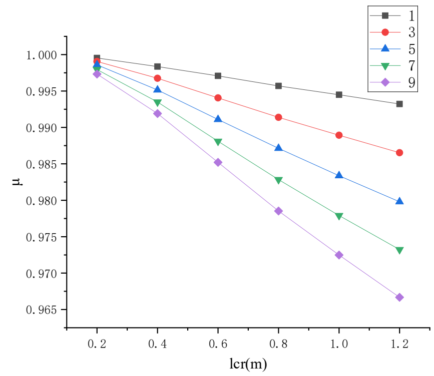


Figure 4: Diagram of the Relationship between the Length of Transverse Cracks and $\boldsymbol{\mu}$.

As can be seen from Figure 4, both an increase in the number of cracks and an increase in crack length will lead to a decrease in μ. By comparing with Figure 3, it is found that the stiffness loss caused by multiple cracks is almost equal to that caused by a single crack. The comparison results are shown in Figure 5 and Figure 6.

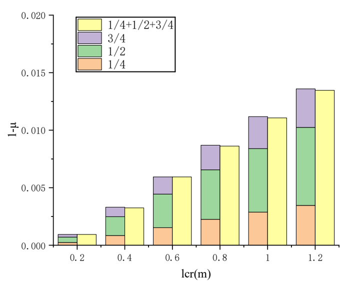


Figure 5: Three Transverse Cracks.

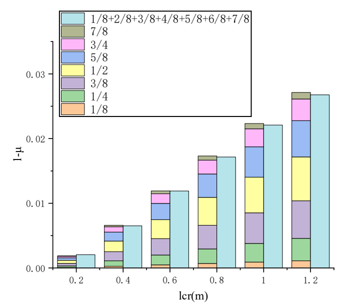


Figure 6: Seven Transverse Cracks.

So the equation of bridge stiffness discount factor under the action of multiple transverse cracks:

$$\mu = 1 - \sum_{i=1}^{n}(1 - \mu_i) \tag{3}$$

μ represents the stiffness reduction coefficient under the joint action of n cracks; μ_i represents the stiffness reduction coefficient under a single i-th crack.

3. Relationship Between Crack Parameters and Stiffness Reduction Coefficient

Through experimental research in reference [6, 7], the stiffness state of a simply - supported bridge after cracks occur is shown in Figure 7. The shaded area represents the stiffness loss due to cracks, presenting a stepped pattern. It can be seen from this that cracks in a bridge always occur near the mid - span area. Therefore, in this paper, the relevant parameters at the mid - span are taken for the study of bridge stiffness.

According to the bond-slip theory in the principle of structural design, the calculation formula for crack width is equation (4), and the crack spacing (the length of the crack influence range) is equation (5). When there is only one crack at the mid-span, it can be calculated by equation (5). When the bending moment during detection cannot be determined, the bending moment under self-weight can be taken.

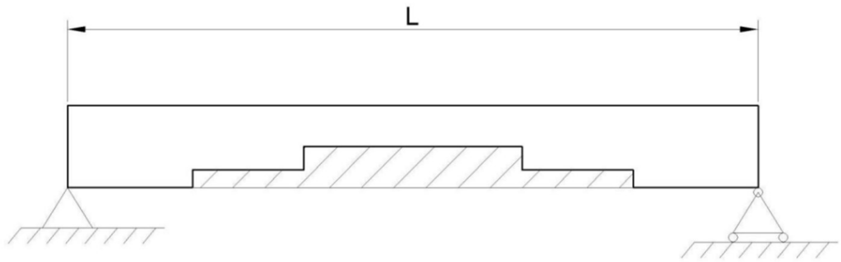


Figure 7: Diagram of Stiffness Loss.

$$\boldsymbol{\omega_{cr} = l_s(\varepsilon_s - \varepsilon_c)} \quad (4)$$

$$\boldsymbol{l_s = \frac{\omega_{cr} I_c E_s E_c}{M(h_m - a_s)(E_c - E_s)}} \quad (5)$$

M represents the bending moment at the crack during bridge inspection; l_s represents the length of the crack influence range; a_s represents the thickness of the steel bar protection layer; ε_c represents the concrete strain near the steel bar at the crack; $\varepsilon_s$represents the steel bar strain at the crack; E_s and E_c respectively represent the elastic modulus of steel bars and concrete.

Through finite element analysis, it was found that when cracks occur, the stress loss at the cracks is approximately triangular, as shown in Figure 7. Therefore, through analysis, when the bridge is under load, the stiffness within the triangular area defined by the length of the crack - affected zone and the crack height has already been lost. Consequently, the stiffness reduction coefficient caused by the cracks should be as shown in the equation (6).

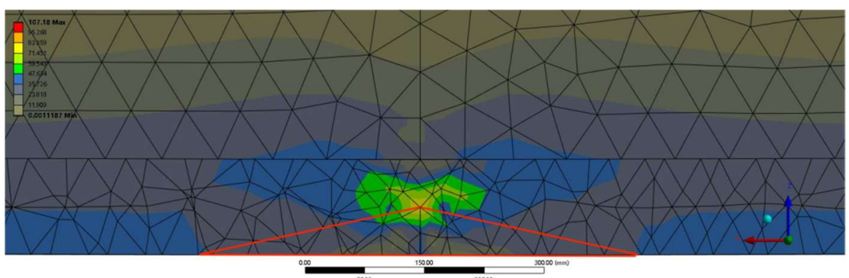


Figure 8: Normal Stress Diagram at the Crack.

$$\boldsymbol{\mu = 1 - \frac{2\int_0^{\frac{l_s}{2}} \left(\frac{l_{cr}\left(\frac{2h_{cr}}{l_s}x\right)^3}{12} + \left(h_m - \left(\frac{h_{cr}}{l_s}x\right)\right)^2 l_{cr}\frac{2h_{cr}}{l_s}x \right) dx}{I_c L}} \quad (6)$$

h_{cr} represents the height (depth) of the crack; l_{cr} represents the length of the crack; h_m represents the height from the bottom plate to the centroid.

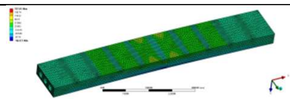
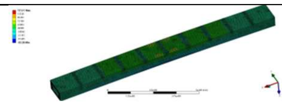
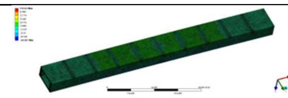
And by incorporating equation (3), we can derive the relationship formula between the stiffness reduction coefficient and crack characteristics:

$$\mu = 1 - \sum_{i=1}^{n} \frac{l_{cri} h_{cri} l_{si}}{I_c L}\left(\frac{{h_{cri}}^2}{12} + \frac{{h_m}^2}{2} - \frac{h_m h_{cri}}{3}\right) \quad (7)$$

l_{cri}, h_{cri} and l_{si} represent the values of the i-th crack respectively; n represents the number of cracks.

In this paper, by simulating the bottom - slab cracks of three types of hollow - slab simply - supported beams, with a load of 1000 kN at the mid - span, the stiffness reduction coefficients are calculated respectively through the fundamental frequency and the formulas derived in this section. The results are shown in Table 2, and the average error rate is 1.43%.

Table 2: Comparison Table of Stiffness Reduction Coefficients for Three Types of Bridges.

	8m hollow core slab	13m hollow core slab	16m hollow core slab
Corresponding force diagram for the base plate method			
Based on finite element calculation values	0.897	0.931	0.967
Calculated values based on the equations in this paper	0.873	0.922	0.945
Error rate	2.13%	0.99%	1.19%

4. Conclusions

This study conducts an in - depth analysis of the relationship between crack parameters and the stiffness reduction coefficient of hollow - slab beam bridges, and the following important conclusions are drawn:

Influence rules of crack parameters on the stiffness reduction coefficient:

The influence rules of crack length, height, position, and quantity on the stiffness reduction coefficient are clarified. An increase in crack length and height leads to a decrease in the stiffness reduction coefficient, and the impact of transverse cracks is more prominent. Cracks at the mid - span have a greater impact on the stiffness reduction coefficient than those at the side - span. Under the action of multiple transverse cracks, the stiffness loss is similar to that caused by a single crack. Corresponding calculation formulas are obtained, providing a key quantitative basis for evaluating the stiffness change of bridge structures under the influence of cracks. This is helpful for accurately judging the attenuation degree of the bearing capacity of bridges.

Derivation and verification of the stiffness reduction coefficient relationship:

Based on the principles of structural design and finite - element analysis, a relationship formula for the stiffness reduction coefficient is successfully derived, which includes parameters such as crack width, spacing, and the crack influence range. Through simulation calculations and verification of three types of 8m, 13m, and 16m hollow - slab simply - supported beams, the average error between the calculation results of this relationship formula and the finite - element calculation values is only 1.43%. This indicates that it has high accuracy and reliability. It can be effectively applied to the performance evaluation and maintenance decision - making of concrete hollow - slab beam bridges in practical engineering. It provides strong support for bridge management departments to reasonably arrange inspection and maintenance plans, ensuring the safe operation of bridges during their service life, reducing the risk of safety accidents, and minimizing unnecessary economic losses.

References

[1] D.K. Agarwalla, D.R. Parhi, Effect of Crack on Modal Parameters of a Cantilever Beam Subjected to Vibration, Procedia Engineering, Volume 51,2013, Pages 665-669. https://doi.org/10.1016/j.proeng.2013.01.094

[2] Zhou, Y., Zhao, Y., Yao, H., Jing, Y., 2019. Full-Scale Experimental Investigation of the Static and Dynamic Stiffness of Prestressed Concrete Girders. Shock and Vibration 2019, 1–13.

https://doi.org/10.1155/2019/7646094

[3] K. Vigneshwaran, R.K. Behera, Vibration Analysis of a Simply Supported Beam with Multiple Breathing Cracks, Procedia Engineering, Volume 86, 2014, Pages 835-842. https://doi.org/10.1016/j.proeng.2014.11.104

[4] Yuwana Sanjaya, Aditya Rio Prabowo, Fitrian Imaduddin, Nur Azmah Binti Nordin,Design and Analysis of Mesh Size Subjected to Wheel Rim Convergence Using Finite Element Method, Procedia Structural Integrity, Volume 33, 2021, Pages 51-58, https://doi.org/10.1016/j.prostr.2021.10.008

[5] Cakram Yudhifa Ganda Satriawan, Widyanita Harwijayanti, Ridwan Ridwan, Aditya Rio Prabowo, Aprianur Fajri, Joung Hyung Cho, Quang Thang Do,Nonlinear analysis of an idealized I-beam member: An investigation of mesh size on the structural behaviors using FE approach,Procedia Structural Integrity, Volume 48, 2023, Pages 50-57, https://doi.org/10.1016/j.prostr.2023.07.109

[6] Zhirong Ma, Xiaoguang Wu, Xingke Chang, Jiang Huang. Concrete Bridge Bearing Capacity Degradation Based on Crack Patterns. Journal of Highway and Transportation Research and Development. 2024, 41(8): 154-161, https://doi.org/10.3969/j.issn.1002-0268.2024.08.016

[7] Ruolin Qian, Pei Su. Calculation of Lateral Distribution of In-Service Bridges Based on Crack Parameters. Journal of Shijiazhuang Tiedao University(Natural Science Edition), 2022,35(02):10-14. 10.3969/j.issn.1002-0268.2024.08.016