The Collaborative Optimization of High-Speed Rail Ticket Pricing and Capacity Considering Passenger Classification

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Abstract: Based on RP (Revealed Preference) and SP (Stated Preference) survey data, this paper utilizes the latent class model to segment high-speed rail passengers, identifying their preferences for different service attributes of the train, such as travel time, departure periods, and ticket prices, and quantifying these preferences. By incorporating revenue management principles and aiming to maximize the overall revenue from multiple trains, a collaborative optimization model for high-speed rail ticket pricing and capacity is developed. The simulated annealing algorithm is employed to solve the model. Finally, a case study is conducted using the high-speed rail route from Xi'an North to Changsha South. The results indicate that, compared to the traditional fixed ticket pricing model, the proposed optimization scheme effectively increases the total railway ticket revenue and provides a feasible reference for the formulation of a flexible pricing mechanism for high-speed rail tickets.

Keywords: High-Speed Rail; Latent Class Model; Dynamic Pricing; Ticket Allocation; Simulated Annealing Algorithm

1. Introduction

Currently, different high-speed rail (HSR) trains operating between the same origin-destination (OD) pair charge uniform fares for the same seat class. This fails to reflect differences in train attributes such as travel time and departure time windows, nor does it account for passengers' preferences for different travel products. Consequently, a supply–demand imbalance persists for HSR passenger products between the same OD pair: some trains experience capacity shortages while others suffer from underutilization, constraining revenue growth.

YAO et al. [1] theoretically analyzed the relationship between market share and fare, developed a nested logit choice model based on stated preference (SP) data, and formulated a pricing strategy for the Wuhan–Guangzhou HSR line. ZHENG et al. [2] adopted a pricing (interval-based) strategy, established a fare optimization model, and explored the properties of optimal multi-level fares. YIN et al. [3] developed a model aimed at maximizing train revenue, investigating the joint optimization of HSR ticket pricing and seat allocation incorporating dynamic passenger demand characteristics during the booking horizon. KIM et al. [4] constructed a direct demand estimation model based on a gravity model to analyze the determinants and impacts of rail fares and travel time. ARMSTRONG et al. [5] reviewed revenue management methods in passenger transportation and suggested that future research should focus on integrating choice models with pricing and allocation schemes. SONG et al.

[6] developed a joint optimization model for seat allocation and ticket pricing under demand uncertainty for a single train, with the objective of maximizing ticket revenue, and solved the model using robust optimization methods. QIN et al. [7] proposed an innovative joint optimization model for fares and seat allocation that incorporates price-elastic demand, finding that joint optimization yields higher revenue.

In terms of passenger behavior analysis, most studies employ elastic demand to describe changes in passenger demand, yet few address the heterogeneity among different passenger types. LI et al. [8] considered passenger heterogeneity and competitive factors, constructing a bi-level model using the epsilon-constraint method. LI et al. [9] applied demand elasticity theory to analyze the elasticities of income, speed, fare, and other factors, offering recommendations for HSR operations management. YANG et al. [10] formulated a pricing strategy based on passengers' value of time and analyzed their sensitivity to fares. HU et al. [11] classified trains using clustering and feature analysis and developed a joint optimization model for class-based ticket pricing and seat allocation.

Existing studies mainly focus on a single passenger type or treat seat allocation and ticket pricing optimization separately. However, different passenger types exhibit diverse preferences and demands, and the level of HSR service quality and fares differentially influence their travel choices. Therefore, it is of great practical significance to comprehensively consider passengers' travel choice behavior and implement differentiated pricing for HSR passenger products between the same OD pair, as this helps balance supply and demand and enhances revenue. In reality, passenger demand is heterogeneous; different passenger groups have varying sensitivities to ticket prices and travel time. Consequently, incorporating passenger classes into the HSR pricing problem can more accurately reflect changes in passenger flows. Demand elasticity coefficients can reasonably characterize this heterogeneity and thereby measure the price sensitivity of different passenger groups.

2. High-Speed Rail Passenger Market Segmentation

Through high-speed rail passenger market segmentation, the differentiated characteristics of passenger choice preferences can be deeply understood. The latent class model (LCM) is employed to implement market segmentation, accurately capturing the probabilistic correlation structure among explicit variables (e.g., age, income level, etc.). This method can provide data support for the optimization of high-speed rail ticket pricing strategies, help operators grasp the differentiated demands of different passenger groups, and targeted optimize the ticket pricing system and ticket allocation, thereby effectively enhancing the competitiveness of passenger transport products.

2.1 Selection and Level Classification of Explicit Variables

Based on the RP and SP survey data of the high-speed rail route from Xi'an North to Changsha South, this paper selects eight indicators as explicit variables: passenger age, monthly income, source of expense, train travel time, expected departure period, ticket price level, and the most valued factor. This transportation survey was conducted at Xi'an North Station, targeting outbound passengers traveling from Xi'an to Changsha South. A total of 380 questionnaires were collected, of which 365 were valid. Each explicit variable is divided into levels, with different numerical values representing the corresponding levels. The specific classification results are shown in Table 1. This paper employs Mplus software to perform latent class analysis, in which the level classification of travel time is primarily based on the number of trains stops and travel speed.

Table 1: Level Classification of Explicit Variables.

Explicit Variable	Symbol	Level	Level Description
Age	U_1	1	<18 years
		2	(18,25] years
		3	(25,45] years
		4	(45,60] years
		5	>60 years
Monthly Income	U_2	1	<5000 CNY
		2	(5000,10000] CNY
		3	(10000,15000] CNY
		4	>15000 CNY
Source of Expense	U_3	1	Business
		2	Self-paid
Train Travel Time	U_4	1	Short [258,338] min
		2	Moderate (338,356] min
		3	Long (356,380] min
Expected Departure Period	U_5	1	Before 08:00
		2	[08:00,12:00)
		3	[12:00,16:00)
		4	16:00 and after
Ticket Price Level	U_6	1	120% of original price
		2	110% of original price
		3	Original price
		4	90% of original price
		5	80% of original price
Most Valued Factor	U_7	1	Economy
		2	Speed
		3	Convenience
		4	Comfort

2.2 Model Fit and Parameter Estimation

To determine the optimal number of latent classes, four models with different numbers of classes were fitted. The Pearson chi-square test ( ), likelihood ratio chi-square test ( ), as well as the AIC and BIC information criteria were selected to evaluate the model fit, where smaller values of each index indicate a better fit. The output indices of each model are shown in Table 2.

Table 2: Model Goodness-of-fit Test Results.

Number of latent classes	AIC	BIC	χ^2	G^2
1	4862.498	4925.613	2879.451	1009.351
2	4620.745	4782.613	1906.423	816.718
3	4512.433	4771.023	1623.159	699.325
4	4508.612	4825.456	1523.946	651.024

From the model fitting results under different numbers of latent classes, it can be seen that as the number of latent classes increases from 1 to 3, the AIC, BIC, and the two chi-square indices all show a gradual downward trend, reaching a relatively optimal level when the number of classes is 3. When the number of classes is further increased to 4, although χ^2 and G^2 indices still decrease slightly, the BIC value rises significantly, indicating that continuing to increase the number of classes would lead to overfitting and a reduction in generalization ability. Based on a comprehensive assessment of the various fit indices, setting the number of latent classes to 3 yields the optimal model. Parameter estimation based on this optimal model yields the proportions of latent classes and the conditional probabilities of each explicit variable. The results are shown in Table 3.

Table 3: Model Parameter Estimation Results.

Explicit Variable	Level	Conditional probability		
		Class 1	Class 2	Class 3
U_1 (Age)	1	0.000	0.055	0.015
	2	0.265	0.810	0.500
	3	0.638	0.118	0.410
	4	0.080	0.018	0.075
	5	0.017	0.000	0.000
U_2 (Monthly Income)	1	0.001	0.884	0.077
	2	0.499	0.116	0.446
	3	0.389	0.000	0.360
	4	0.111	0.000	0.116
U_3 (Source of Expense)	1	0.134	0.025	0.873
	2	0.866	0.975	0.129
U_4 (Train Travel Time)	1	0.767	0.082	0.256
	2	0.219	0.298	0.435
	3	0.014	0.620	0.309
U_5 (Expected Departure Period)	1	0.060	0.124	0.039
	2	0.593	0.397	0.368
	3	0.165	0.286	0.322
	4	0.182	0.183	0.271
U_6 (Ticket Price)	1	0.751	0.000	0.142
	2	0.237	0.000	0.528
	3	0.012	0.138	0.322
	4	0.000	0.379	0.008
	5	0.000	0.483	0.000
U_7 (Most Valued Factor)	1	0.332	0.569	0.391
	2	0.326	0.214	0.450
	3	0.190	0.129	0.079
	4	0.152	0.088	0.080
Latent class probability		0.420	0.220	0.360

From the latent class probabilities shown in Table 3, it can be seen that Class 1 passengers account for the highest proportion, with a probability of 0.42; Class 3 passengers rank second, with a probability of 0.36; and Class 2 passengers account for the smallest proportion, with a probability of 0.22. From the conditional probability distribution, it is found that passengers in different classes have significant differences in the characteristics of explicit variables, particularly in the three indicators of train travel time, expected departure period, and ticket price level, indicating that these three explicit variables are the key influencing factors in this passenger classification.

2.3 Latent Class Results

Based on the differences in conditional probabilities of each latent class on the explicit variables, the travel preferences and choice characteristics of different passenger groups can be extracted. From Table 3, it can be seen that: Class 1 passengers prefer trains with shorter travel times and morning departures, have average requirements for comfort, and are insensitive to ticket prices; Class 2 passengers can accept longer travel times and general departure periods, and prioritize ticket price levels; Class 3 passengers have higher requirements for departure periods and ride comfort, have prominent time value, and have relatively low sensitivity to ticket prices. Accordingly, the three classes of passengers are respectively named: leisure passengers, economy passengers, and business passengers. Combined with the questionnaire data, the train choice patterns and demand differences of various passenger classes can be further identified.

3. Problem Description and Assumptions

3.1 Problem Description

This paper takes a specific high-speed rail line as the research object. The latent class model is used to segment the passenger market and quantify the preference characteristics of different passenger groups regarding ticket price levels, departure times, and travel time. Combined with the Logit model, passenger flow assignment is completed. Finally, with the core objective of maximizing the revenue of the high-speed rail operating unit, and considering constraints such as train capacity and upper and lower ticket price limits, a collaborative optimization model for high-speed rail dynamic pricing and ticket allocation that accounts for passenger classification heterogeneity is constructed. The model is then solved to obtain the optimal ticket price levels and corresponding ticket allocation strategies for each OD pair, providing theoretical support and practical guidance for high-speed rail operation optimization.

3.2 Assumptions

(1) Only second-class seats are considered, and adjustment behaviors in the ticketing process such as ticket refunds and exchanges are not taken into account.

(2) The weights assigned to train service attributes by passengers of the same type remain consistent across different OD pairs.

4. Collaborative Optimization Model for High-Speed Rail Train Ticket Pricing and Capacity

4.1 Analysis of Elastic Passenger Travel Demand

Passenger demand is a macro-level manifestation of travel decisions. In the study of collaborative optimization for high-speed rail dynamic pricing and ticket allocation, accurately characterizing the elastic demand characteristics of passengers (e.g., their sensitivity to changes in price, time, service, and other factors) is key to the success of the model. The high-speed rail passenger demand in any given time period can be expressed as an elastic function of the generalized travel cost of passengers during that period. The generalized travel cost of high-speed rail passengers mainly consists of three components: ticket price, travel time cost, and expected departure period deviation cost. That is, when a passenger chooses train h between OD pair (r,s), the generalized travel cost is:

$$c_{rs}^{h} = p_{rs}^{h} + \omega_1 \cdot t_{rs}^{h} + \omega_2 \cdot \left|f_{rs}^{h} - d_{rs}^{h}\right| \tag{1}$$

In the equation: p_{rs}^{h} is the ticket price for train h between (r,s); t_{rs}^{h} is the travel time of train h between (r,s), f_{rs}^{h} is the departure time of train h between (r,s), d_{rs}^{h} is the expected departure time of train h between (r,s) for the passenger; ω_1 is the average unit time value of the passenger's travel time, ω_2 is the average unit time value of the passenger's expected departure time deviation. The elastic passenger demand of class m on OD pair (r,s) can be described as:

$$q_{rs}^{hm}(c_{rs}) = a_{rs}^{hm} \cdot q_{rs}^{o}(c_{rs}^{o}) \cdot exp\left[-\eta_m\left(\frac{c_{rs}^{m}}{c_{rs}^{om}} - 1\right)\right] \tag{2}$$

Using the Logit model for passenger flow assignment, the probability that a class m passenger on OD pair (r,s) chooses train h for travel can be calculated, and its expression is as follows (where θ is the scaling parameter):

$$P_{rs}^{hm} = \frac{exp\left(-\theta C_{rs}^{hm}\right)}{\sum_{g=1}^{n} exp\left(-\theta C_{grs}^{m}\right)} \tag{3}$$

4.2 Model Formulation

To facilitate the subsequent description, the symbols and variables are defined in Table 4.

Table 4: Symbols and Variable Definitions.

Category	Symbol	Description
Sets	W	The set of OD pairs in the passenger transportation network formed by multiple trains.,OD pairs $(r,s) \in W$
	L	segment set，$l \in L$
	H	The set of all trains departing on a certain day, the set of trains between(r,s), $H_{rs} \subset H$, $h \in H$
Parameters	ω_i	Average value of unit time
	C_h	Train h seating capacity
	a_{rs}^{m}	Proportion of class m passengers between(r,s)
	M	Types of passengers
Variables	$\bar{p}_{rs}^{h}$	The fare ceiling for train h between (r,s)
	$\underline{p}_{rs}^{h}$	The fare floor for train h between (r,s)
	t_{rs}^{h}	The travel time of train h between (r,s)
	q_{rs}^{h}	The elastic passenger flow of train h between (r,s)
	c_{rs}^{hm}	Generalized travel cost for a class m passenger traveling on (r,s)choosing train h
Variables	x_h^n	0-1 variable: When train h stops at station n, the value is $x_h^n = 1$; otherwise, it is $x_h^n = 0$
Decision variables	p_{rs}^{h}	The ticket price of train h between (r,s)
	b_{rs}^{h}	The ticket quota of train h between (r,s)

4.3 Objective Function

This model aims to maximize the total high-speed rail ticket revenue R, that is:

$$max\, R = \sum_{h\in H} \sum_{(r,s)\in W} p_{rs}^{h} \cdot b_{rs}^{h} \tag{4}$$

4.4 Constraints

The model simultaneously satisfies the following constraints:

(1) Train capacity constraint:

$$\sum_{r=1}^{j} \sum_{s=j+1}^{N} b_{rs}^{h} \leq C_h, \forall h \in H, \forall j \in [1, N-1] \tag{5}$$

(2) Transport service accessibility constraint:

$$(x_h^r \cdot x_h^s - 1) \cdot b_{rs}^{h} = 0, (r,s) \in W, h \in H \tag{6}$$

(3) Ticket allocation not exceeding demand constraint:

$$b_{rs}^{h} \leq q_{rs}^{h}, (r,s) \in W, h \in H \tag{7}$$

(4) Upper and lower bound constraints on ticket price:

$$\underline{p}_{rs}^{h} \leq p_{rs}^{h} \leq \bar{p}_{rs}^{h}, (r,s) \in W, h \in H \tag{8}$$

(5) Ticket price mileage constraint:

$$If\ l_{mn} > l_{rs}, then\ p_{mn} > p_{rs}, \forall (m,n), (r,s) \in W \tag{9}$$

(6) Integer constraint:

$$p_{rs}^{h} \in N, b_{rs}^{h} \in N, (r,s) \in W, h \in H \tag{10}$$

5. Algorithm Design

The collaborative optimization model for high-speed rail ticket pricing and ticket allocation constructed in this chapter is a typical nonlinear integer programming problem. Based on the characteristics of the problem and the structural features of the model, this paper selects the simulated annealing algorithm for solution design. The specific steps of the simulated annealing algorithm are as follows:

(1) Initial parameter settings: Set the initial temperature as T_0, the termination temperature as T_{end}, and the cooling coefficient as φ. Under the constraints (8) and (9), randomly generate the initial ticket pricing scheme for each train S_0. Let $T = T_0$, $S = S_0$ and set the upper limit of iterations for each time step T to U_{max}. Let the current temperature be $U_s = 0$, the temperature step decrease be $Q = 0$, and the current optimal solution be $S_{best} = S_0$.

(2) Based on the ticket pricing solution S and other service level indicators, determine the passenger demand for each train on each OD pair. Use the VS platform to edit the algorithm and solve the model to obtain the optimal ticket allocation plan under this ticket pricing scheme.

(3) Calculate the transfer passenger flow to adjust the passenger demand for each OD pair and each train, and then obtain the optimized total railway ticket revenue R.

(4) Generate a new ticket price decision variable S_1 based on Equation (4) and the current solution S.

(5) If S_1 satisfies the optimization model constraints (5) and (6), then repeat steps 2 and 3 to obtain the difference ΔR between R' and R; otherwise, proceed to step 4.

(6) Use the Metropolis criterion to decide whether to accept the new solution. When $\Delta R < 0$, accept the new solution S_1, meaning $S = S_1$; $\Delta R > 0$, compute the acceptance probability S_1 for $exp(\Delta R/T)$; if $exp(\Delta R/T) > random(0,1)$, set $S = S_1$; otherwise, retain the current solution S.

(7) Compare R with R_{best}. When $R > R_{best}$ (likely referring to a comparison between the two

values), set $S_{best} = S, pb = 0$; otherwise, retain the current optimal solution S_{best}.

(8) $U_s = U_s + 1$, when $U_s > U_{max}$, end this state, perform $U_l = 0$ and proceed to step 8; otherwise, proceed to step 4.

(9) Cooling down.$T_{Q+1} = e^{-\varphi} T_Q, T = T_{Q+1}, Q = Q + 1$.

(10) Terminate the check. When $T < T_{end}$, output the current optimal solution S_{best} and the corresponding R_{best}, and end the program; otherwise, proceed to step 4.

6. Case Study Analysis

6.1 Case Study Background

Nine high-speed trains operating from Xi'an North to Changsha South, namely G520, G824, G866, G828, G844, G816, G832, G1166, and G848, are selected as the research objects for calculation and analysis, with train numbers set from 1 to 9 in sequence. This line has 11 stopping stations and 10 sections, totaling 55 OD pairs. The stopping plans for each train are shown in Figure 1. The passenger demand and initial ticket price for each OD pair are based on actual ticket sales data. The upper and lower bounds for ticket price fluctuation are set at 120% and 80% of the original ticket price of the corresponding OD pair, respectively.

The parameters for the simulated annealing algorithm are set as follows: The initial temperature is set to 1000 degrees, and the termination temperature is set to 0.1 degrees. The upper limit for the number of iterations at each temperature is set to 10, and the cooling rate is set to 0.987. The algorithm terminates if the temperature falls below the termination temperature or if there is no change in the objective function after 200 consecutive iterations.

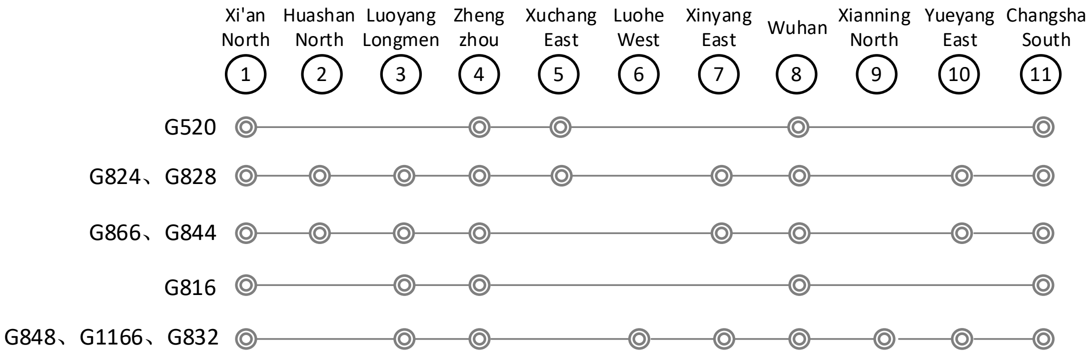

Figure 2: Iteration Curve of the Objective Function

Based on the optimal objective function value obtained from the convergence of the above algorithm, this paper selects typical OD pairs to analyze the results of ticket pricing and ticket allocation. The case study includes a total of 55 OD pairs. To highlight the typical characteristics of the optimization results, this paper screens 15 representative OD pairs from multiple dimensions, including travel distance, location, train service coverage, and travel demand, to present the ticket pricing and ticket allocation schemes, thereby comprehensively reflecting the applicability of the model in different scenarios. The ticket pricing scheme for trains on the selected OD pairs is shown in Table 5.

Table 5: Fare Scheme Under Optimal Revenue (Unit: Yuan).

OD	G520	G824	G844	G816	G832	G1166
1-11	630.6	627.6	551.3	697.7	724.3	499.5

1-8	522.7	486.3	450.7	503.5	547.7	450.0
4-11	434.4	439.2	385.9	470.9	488.9	366.8
1-7	442.0	423.2	371.8	425.8	442.0	366.8
8-11	216.4	209.2	183.8	205.3	213.1	183.6
1-3	250.2	224.2	196.9	241.0	190.2	192.6
4-7	202.2	187.8	165.0	194.8	192.4	163.8
8-10	93.2	94.7	83.2	92.9	105.2	81.7
3-8	329.7	313.0	275.0	317.6	358.1	272.7
2-4	216.4	191.0	167.8	208.4	190.2	163.8
1-2	59.4	58.3	51.2	57.2	59.4	49.0
4-5	45.2	44.4	39.0	43.6	57.2	39.1
5-6	34.3	33.7	29.6	33.1	34.3	29.2
8-9	40.9	41.2	36.2	40.4	42.0	35.1
10-11	74.7	79.7	70.0	78.2	72.5	63.9

As shown in Table 6, ticket prices vary significantly across different trains and OD pairs, with both increases and decreases coexisting, verifying that the optimization results incorporating passenger classification are reasonable. Compared with the original ticket prices, G816 and G832 generally show an upward trend, with a maximum increase of 9%, while G844 and G1166 are mainly characterized by price reductions. The price differences between fast and slow trains intuitively reflect the value of travel time. Among them, short-haul OD pairs face competition from alternative modes of transport such as highways and intercity railways, resulting in relatively limited pricing space. The ticket allocation results corresponding to the above ticket pricing optimization scheme are shown in Table 6.

Table 6: Seat Allocation Scheme Under Optimal Revenue (Unit: Seats).

OD	G520	G824	G844	G816	G832	G1166
1-11	25	20	15	30	25	15
1-8	45	40	30	55	50	35
4-11	85	90	60	100	95	70
1-7	35	30	25	40	35	20
8-11	120	125	85	135	130	100
1-3	40	35	30	45	40	30
4-7	45	50	35	55	50	40
8-10	110	115	80	125	120	90
3-8	55	60	45	65	60	45
2-4	25	30	20	35	30	20
1-2	55	50	45	60	55	40
4-5	90	95	70	100	95	75
5-6	85	90	65	95	90	70
8-9	90	95	70	100	95	75
10-11	90	95	70	100	95	75

Based on the optimal ticket pricing scheme, the model simultaneously optimizes the ticket allocation for each train across OD pairs. The allocation process fully matches the pricing strategies of different trains with the travel characteristics of target passenger groups. For example, G816, as a premium express train, has the highest ticket price level, and its ticket allocation is tilted toward long-haul OD pairs to ensure the travel demand of time-sensitive business passengers. G1166, as an economy train, has the lowest ticket price, with ticket allocation favoring short-haul OD pairs to attract price-sensitive passenger groups. G844, as a price-reduced slow train, also allocates more tickets to short-haul OD pairs, creating a synergistic effect with the price reduction strategy and further enhancing its appeal to economy passengers. G520 and G824, as conventional trains, have relatively balanced ticket allocation, covering the travel needs of different types of passengers. The total ticket allocation for each train is controlled near its seating capacity, ensuring efficient capacity utilization while avoiding overcrowding, demonstrating the reasonableness and operability of the ticket allocation optimization results.

7. Conclusion

With the further introduction of passenger classification, all core operational indicators have achieved significant improvements: ticket revenue increased from 2,124,033 CNY to 2,267,749 CNY, an increase of approximately 6.76%, indicating that differentiated ticket pricing and allocation strategies targeting different passenger types can more accurately tap revenue potential and achieve refined revenue growth for railway operators. The passenger travel success rate on the core OD pair of Xi'an North–Changsha South increased from 81.25% to 84.03%, an increase of 2.78 percentage points, demonstrating that ticket allocation after passenger classification better matches the travel needs of different groups, effectively reducing demand loss and improving passenger travel experience. The average train occupancy rate increased from 66.59% to 73.29%, an increase of 6.70 percentage points, reflecting more efficient capacity resource allocation after classification optimization, fuller utilization of train capacity, and alleviation of the contradiction between idle capacity during off-peak periods and insufficient capacity during peak periods. Overall, the collaborative optimization scheme incorporating passenger classification further achieves dual improvements in revenue and capacity utilization while maintaining travel service levels, verifying the effectiveness and necessity of passenger classification modeling in the collaborative optimization of high-speed rail ticket pricing and allocation.

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